Billy Lam
OPT 425
(*) - may not be correct.
Constants:
λ 30 $\mu m - 3mm$ (THz)
$0.7\mu - 30\mu m$ (IR)
0.4-0.7 μm (visible)
$\lambda \leq 0.4\mu m$ (UV)
$K_m = 683\frac{lm}{W}$
$\sigma = 5.67 \cdot 10^{-8}\frac{W}{m^2K^4}$
$\lambda_{max} \approx \frac{2898\mu mK}{T}$ for L_λ
$\nu_{max} = 5.78 \cdot 10^{10}\frac{Hz}{K}T$

Photon-based radiaometric Quantities of BB
$u_{p,\nu} = \frac{8\pi v^2}{c^3}\frac{1}{e^{\frac{h\nu}{kT}}-1}$
$L_{p,\nu} = \frac{2\nu^2}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$
$M_{p,\nu} = \frac{2\pi\nu^2}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$
$L_{p,\lambda} = \frac{2c}{\lambda^4}\frac{1}{e^{\frac{h\nu}{kT}}-1}$
$M_{p,\nu} = \frac{2\pi c}{\lambda^4}\frac{1}{e^{\frac{h\nu}{kT}}-1}$
$\nu^p_{max} = 3.315 \cdot 10^{10}\frac{Hz}{K}T$
$\lambda^p_{max} = \frac{3675\mu mK}{T}$

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human eye characteristic
pupil-size : $2 - 8mm$
lens to retina $20mm$
size of photoreceptor $\sim 3\mu m$
$n_{eye} = 1.336$
threshold luminance luminance
Photopic - $10^{-3}\frac{cd}{m^2}$
Scotopic - $10^{-6}\frac{cd}{m^2}$
Angular resolution $\sim$ 1 minute of arc)
Temporal resolution $\sim$ 20 msec
need to know:
(Lambertian disk source with small detector) $\frac{d\Phi}{dA_S} = M = \pi L \sin^2\theta_{\frac{1}{2}}$

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general parameters - light/EM waves $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$
parameters affected by n: λ, k
$\lambda = \frac{\lambda_0}{n}$ λ_0 vacuum value.
$k = nk_0$
parameters unaffected by n (energy, frequency group): $E, \nu, \omega, \tilde{w}$
$\tilde{\nu} = \frac{1}{\lambda_0}$ "wavenumber" (number of wavelength in 1 cm)

$\theta r = l \Rightarrow d\theta = \frac{dl}{r}$
$\Omega = \frac{A}{r^2} \Rightarrow d\Omega = \frac{dA_\perp}{r^2} = \frac{dA\cos\theta}{r^2}$
$\Omega_{sph} = 4\pi$ sr
$dA = r^2 \sin\theta d\theta d\phi$
$d\Omega = \sin\theta d\theta d\phi$
$\Omega_{hemi} = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\theta d\theta d\phi = 2\pi$

Radiometric quantities - energy based

Energy	Q		*joule*	in a beam of N photon, $Q = Nh\nu$
Energy density	u	$\frac{dQ}{dV}$	J/m^3	
Flux	Φ	$\frac{dQ}{dt}$	watt	optical power
Flux density		$\frac{dE}{dA}$	$\frac{\text{watt}}{cm^2}$	
Radiant exitance	M	$\frac{d\Phi}{dA_\perp}$	$\frac{\text{watt}}{cm^2}$	leaving a surface
Irradiance	E			incident on the surace
Radiances	L	$\frac{d^2\Phi}{dA_\perp d\Omega}$	$\frac{\text{watt}}{cm^2(steradian)}$	$\Omega =$ solid angle
intensity	I	$\frac{d\Phi}{d\Omega}$	$\frac{\text{watt}}{(steradian)}$	

Photon based quantities

photons	Q_P		phtons	
photon density	u_p	$\frac{dQ_P}{dv}$	photons/m^3	
photon Flux	Φ_P	$\frac{dQ_P}{dt}$	photons/s	
photon flux densities	E	$\frac{d\Phi_P}{dV?}$	$\frac{\text{photons}}{cm^{3?}}$	
photon Flux Extence	M_P	$\frac{d\Phi_P}{dA}$	$\frac{\text{photons}}{cm^2}$	
Incident photon flux density	E_P			
Photon Flux Radiance	L	$\frac{d^2\Phi_P}{dA_\perp d\Omega}$	$\frac{\text{photons}}{cm^2(steradiaan)}$	
Photon Flux Intensity	I_P	$\frac{d\Phi}{d\Omega}$	$\frac{\text{photons}}{(steradiaan)}$	photon/s(solid angle)

Photometric Quantities

name	Photometrix Quantities	math	unit
Luminance Energy	Q_v		$\frac{\text{units}}{\text{talbot}}$
Luminous Density	u_v	$\frac{dQ}{dv}$	$\frac{\text{talbot}}{m^2}$
Luminous Flux	Φ_v	$\frac{dQ}{dt}$	Lumen(lm)
Luminous Exitance	M_v	$\frac{\Phi_v}{dA}$	$\frac{lm}{m^2} =$ Lux
Illuminance	E_v	$\frac{\Phi_v}{dA}$	$\frac{lm}{m^2} =$ Lux
Luminance	L_V	$\frac{d^2\Phi}{dA_\perp d\Omega}$	$\frac{lm}{m^2 \cdot sr.}$
Luminance Intensity	I_v	$\frac{d\Phi_v}{d\Omega}$	$\frac{lm}{sr.}$

$\Phi_\lambda = \left|\frac{d\Phi(\lambda)}{d\lambda}\right|, \Phi_\nu, \Phi_{h\nu}$

Convert by multiplying by $\left|\frac{d\lambda}{d\nu}\right|$

$\Phi = \int_{\lambda_1}^{\lambda_2} \Phi_\lambda(\lambda) d\lambda \qquad$ (reflected: $R(\lambda)$, transmitted: $T(\lambda)$)

$\Phi_p = \int \Phi_\lambda(\lambda) \frac{\lambda}{hc} d\lambda$

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point source - emits uniformly in all direction

$I = \frac{d\Phi}{d\Omega} \frac{\Phi}{4\pi}$

$\Phi_d = \int_{A_d} d\Phi_d$

$\frac{\Phi}{4\pi} \cdot d\Omega_S = d\Phi_d$

$\frac{I_S dA_d}{r^2} = d\Phi_d$

fraction $\frac{dA_d}{4\pi r^2}$

power of $A_d = \Phi \frac{dA_d}{4\pi r^2}$

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detector inclined

dist. to screen $= z$ $d\Phi_d = \frac{\Phi_d}{4\pi} \frac{dA_d \cos\theta}{r^2} = \frac{\Phi_d}{4\pi} \frac{dA_d \cos^3\theta}{z^2}$

$dE_d = \frac{d\Phi_d}{dA_d} = \frac{\Phi_d}{4\pi} \frac{\cos^3\theta}{z^2}$

$$\frac{L_1}{n_1^2} = \frac{L_2}{n_2^2}$$

Extended source differential source source and detector area.

$da_s, dA_d, d^2\Phi_d$

$\Phi_d =$ total power from $A_s \to A_d$

$d^2\Phi_s =$ power leaving dA_s heading to dA_d

$d^2\Phi_d =$ power arriving dA_d from dA_s

$d\Phi_s = \int_{\text{all direction source emits}} d^2\Phi_s$

$d\Phi_d = \int_A d^2\Phi_{s \cup d?}$

–

extended sourece: $L_S = \frac{d^2\Phi_S}{dA_\perp d\Omega_S}$

$L_S dA_\perp d\Omega_S = d^2\Phi_S = d^2\Phi_d$

$L_S dA_S \frac{dA_d}{r^2} = d^2\Phi_d$

$\frac{L_S dA_S}{r^2} = d\Phi_d$

–

source tilted.

$L_S dA_{S,\perp} d\Omega_S = d^2\Phi_d = L_S dA_S \cos\theta \frac{dA_d}{r^2} = d^2\Phi_d$

$\frac{L_S dA_S \cos\theta}{r^2} = dE_d$

Lambert cosine law

$d\Phi_d \cos\theta$

$d^2\Phi_d = \frac{L_s dA_S \cos\theta d\Omega_S}{r^2}$

law holds if L_S independent of angle θ (Lambertian source (wall, paper, ...).

$d^2\Phi_d = LdA_s \cos\theta_S \frac{dA_d \cos\theta_d}{r^2}$

Weak form of Radiance Theorem
$L_S = \frac{d^2\Phi_S}{dA_{S,\perp} d\Omega_S} = \frac{d^2\Phi_S}{dA_\perp \cos\theta_S dA_d \cos\theta_d / r^2}$
$L_D = \frac{d^2\Phi_d}{dA_\perp \cos\theta_S dA_d \cos\theta_d / r^2}$
$d^2\Phi_S = d^2\Phi_d$ fpr lossless propagation
if there's loss, there usually is some parameter that tells us how much loss (1-R - transmission of glass, R = loss, absorption from air, A = loss)
$L_d = TL_S$
relationship between L_S and $\Phi_S, A, M, \frac{\Phi_D}{A_S} = M$
–
$L_S = L_d$
2 standpoints: source standpoint - $d^2\Phi_S (= d^2\Phi_d) = L_S dA_S \cos\theta_S d\Omega_S$
detector stand point $d^2\Phi_d = L_d dA_d \cos\theta_d d\Omega_d$
–
Lambertian shource emits in 2π sr.
$d\Phi_S = \int_\Omega L_S dA_s \cos\theta d\Omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} L_S dA_s \sin\theta \cos\theta d\theta d\phi = \pi L_S dA_S$
$M = \pi L_S$

$$d\Phi_d = \pi L dA_S \sin^2\theta_{\frac{1}{2}}$$

(Lambertian source emiting into cone of half angle $\theta_{\frac{1}{2}}$)

$$M = \pi L \sin^2\theta_{\frac{1}{2}}$$

Tilted source, tilted detector.
Solid angle are small $A << r^2$
integrate for big source and//or detector.
$d^2\Phi_d = LdA_S \cos\theta_S \frac{dA_d \cos\theta_d}{r^2}$
$\cos^4$ Law: $\frac{d^2\Phi_d}{dA_d} = E = \frac{LdA\cos^2\theta}{(\frac{z}{\cos\theta})^2} = \frac{LdA\cos^4\theta}{z^2}$
–
single integration of differential expression

Big source, small detector

small source, big detector

point source $d\Phi_d = Id\Omega_S$
$\Phi_d = I\int \Omega_S = I\Omega$
for disk normal to point source (S on disk axis)
–
extended detector or source

Lambertian disk source, normal on axis small detector.

$\left(A_S, d\rho, \rho d\phi, dA = \rho d\rho d\phi, d^2\Phi = LdA_S \cos\theta \frac{dA_d \cos theta}{r^2}\right), d\Phi = \int_{A_S} d^2\Phi$
source standpoint:

$$d\Phi = \int_0^{2\pi} \int_0^{\theta_{\frac{1}{2}}} L \underbrace{z\tan\theta}_{\rho} \underbrace{z\frac{d\theta}{\cos^2\theta}}_{d\rho} d\phi \cos\theta \frac{dA_d \cos^3\theta}{z^2}$$

$$
\begin{aligned}
&= L\int_0^{2\pi}\int_0^{\theta_{\frac{1}{2}}} d\theta d\phi \sin\theta\cos\theta \\
&= LdA_d \cdot 2\pi \cdot \frac{\sin^2\theta_{\frac{1}{2}}}{2} \\
&= LdA_d \cdot \pi \cdot \sin^2\theta_{\frac{1}{2}}
\end{aligned}
$$

Detector standpoint:
$d^2\Phi_d = LdA_d\cos\theta d\Omega_d$

$$
\begin{aligned}
d\Phi_d &= \int_{\Omega_d} LdA_d\cos\theta \underbrace{\sin\theta d\theta d\phi}_{d\Omega_d} \\
&= \int_0^{2\pi}\int_0^{\theta_{\frac{1}{2}}} LdA_d\cos\theta\sin\theta d\theta d\phi
\end{aligned}
$$

$\frac{d\Phi}{dA_d} = E = \pi L\sin^2\theta_{\frac{1}{2}}$

Similar problem
Large disk detector, normal on-axis small lambertian source

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small aperture between (Lambertian) A_s, A_d
$r_{sa}, r_{ad}, \theta_s, \theta_d$(half angle), L_s
weak form of radiance theorem: $L_s = L_a$. Aperature also lambertian source emitting into a cone of half angle $= \theta_s$.
If $\theta_s > \theta_d$, entire detector receiving light (detector overfill).
$d\Phi_d = \pi L\sin^2\theta_d dA_a$
$\max E_d$(on-axis)
$d^2\Phi = LdA_a\frac{dA_d}{r_{ad}^2}$
$E_{max} = \frac{LdA_a}{r_{ad}^2}$
$E_d(\theta) = E_m\cos^4\theta = \frac{LdA_a}{r_{ad}^2}\cos^4\theta$
If $\theta_s < \theta_d$
$d\Phi_d = \pi L\sin^2\theta_s dA_a$

$d^2\Phi = LdA_s\cos\theta_s\frac{dA_d\cos\theta_d}{r^2}$
If the area (source, detector) is greater than $\frac{1}{30}$ of the distance b/w s,d, then we have to take dA_s, dA_d.
$d\Phi = \pi L\sin^2\theta\frac{1}{2}dA_s$
$d\Phi = \pi L\sin^2\theta_{\frac{1}{2}}dA_d$
$\frac{d\Phi}{dA_d} = \pi L\sin^2\theta_{\frac{1}{2}}$

Numerical integration (parallel Lamb source, detector)
$\vec{R}_{ij} = x_i\hat{x} + y_j\hat{y} + z\hat{z}$
$\vec{R}_{ij}\cdot\vec{z} = R_{ij}z\cos\theta$
$z^2 = \sqrt{x_i^2 + y_j^2 + z^2}z\cos\theta$

$\cos\theta = \frac{z}{\sqrt{x_i^2 + y_j^2 + z^2}}$
$d^2\Phi_{ij} = \frac{L dA_s \cos^2\theta dA_d}{R_{ij}^2}$
$= \frac{L dA_s z^2 (\Delta l)^2}{(x_i^2 + y_j^2 + z^2)^2}$
$d\Phi = \sum_i \sum_j \frac{L dA_s z^2 (\Delta l)^2}{(x_i^2 + y_j^2 + z^2)^2} = L dA_s z^2 (\Delta l)^2 \sum_i \sum_j \frac{1}{((i\Delta l)^2 + (j\Delta l)^2 + z^2)^2}$
–
2 numerical integration
$dA_{ab} = (\Delta l)^2$ centered at $(x_a, y_b, 0)$
$\cos\theta_{abij} = \frac{z}{\sqrt{(x_a - x_i)^2 + (y_b - y_j)^2 + z^2}}$
$\Phi = \sum_a \sum_b \sum_i \sum_j \frac{L z^2 (\Delta l)^4}{((x_a - x_i)^2 + (y_b - y_j)^2 + z^2)^2} = L z^2 (\Delta l)^4 \sum_a \sum_b \sum_i \sum_j \frac{1}{((a-i)^2(\Delta l)^2 + (b-j)^2(\Delta l)^2 + z^2)^2}$

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analyzical solution
Lamb source $A_s \parallel A_d$
$\int_{det} \int_{\text{source}} \frac{L dA_s dA_d \cos^2\theta}{r_{sd}^2} = \frac{2L(\pi R_s R_d)^2}{R_s^2 + R_d^2 + z^2 + \sqrt{(R_s^2 + R_d^2 + z^2)^2 - 4R_s^2 R_d^2}}$

Refraction - Radiometry
Lossless Refraction
det. standpoint $d^2\Phi_1 = L_1 dA \cos\theta_1 d\Omega_1$
source standpoint $d^2\Phi_2 = L_2 dA \cos\theta_2 d\Omega_2$
$L_1 dA \cos\theta_1 \sin\theta_1 d\theta_1 d\phi_1 = L_2 dA \cos\theta_2 d \sin\theta_2 d\theta_2 d\phi_2$
$n_1 \sin\theta_1 = n_2 \sin\theta_2$
differential snell's law $n_1 \cos\theta_1 d\theta_1 = n_2 \cos\theta_2 d\theta_2$
substitution and get the strong form of radiance theorem

$$L_1 = \left(\frac{n_1}{n_2}\right)^2 L_2 \qquad \text{or} \qquad \frac{L_1}{n_1^2} = \frac{L_2}{n_2^2}$$

basic radiance is conserved for lossless refraction. (and propagation)

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len between s, d. ϕ - clear diameter.
abbe sign condition $a_1 h_1 \sin\theta_1 = n_2 h_2 \sin\theta_2$
$\Rightarrow m = \frac{n_1 \sin\theta_1}{n_2 \sin\theta_2}$
if $n_1 = n_2$, small $\theta' s$, $m = \frac{\tan\theta_1}{\tan\theta_2} = \frac{s_2}{s_1}$ (paraxial)
Ignore diffraction $h_2 = h_1 \frac{\sin\theta_1}{\sin\theta_2}$
diffraction limited spot size $h_2 = 2.44\lambda f$
use whichever one that's big.
for small $h_1 \frac{\sin\theta_1}{\sin\theta_2}$, diffraction can't be ignored and h_2 would be bigger than $h_1 \frac{\sin\theta_1}{\sin\theta_2}$.
imageside eqs. - don't work
use source side eqs $d\Phi_i = d\Phi_{lens} = \pi L \sin^2\theta_1 dA_s$
overfill - not common
underfill - $d\Phi_d = d\Phi_i = d\Phi_{len} = \pi L \sin^2\theta_1 dA_s = \pi L \frac{(\phi/2)^2}{s_1^2} dA_s$
Case I over fill: $d\Phi_d = E_i dA_d = \pi L \sin^2\theta_2 dA_d$ (indep. of s_1)
As s_1 go from 0 to large. First overfill case $\rightarrow$ underfill or diffraction whichever comes first.
photometry - brightness to human eye

we'll find the same behavior
as object gets farther and farther away $\rightarrow$ to the eye it looks like a point source. cuz illuminating only a few photoreciptor.

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radiometry of images (assume $n_1 = n_2$, lossless, lambertian, no diffraction, differential area)
$d\Phi_{lens} = \pi L \sin^2\theta_1 dA_1 = d\Phi_2$
$\max(\sin\theta_1) \rightarrow \max(\Phi)$ (bigger ϕ, move s closer 2 lens)
$\frac{d\Phi}{dA_2} = \pi L \sin^2\theta_1 \frac{dA_1}{dA_2} = \frac{\pi L}{m^2}$
From image size
$d\Phi_2 = \pi L_l \sin^2\theta_2 dA_2$
$E = \pi L \sin^2\theta_2$ bigger with bigger ϕ, or smaller f
$\sin^2\theta dA_1 = \sin^2\theta_2 dA_2$
$h_1 \sin\theta_1 = h_2 \sin\theta_2$

Power on detector using lens
1) image overfill detector
$E_i = E_d = E = \pi L \sin^2\theta_2$
$\Phi = \pi L \sin^2\theta_2 dA_2$
Image underfill
$d\Phi_1 = d\Phi_d$
$\pi L \sin^2\theta_1 dA_1 = d\Phi_d$
$\max(\Phi)$ when $s_2 = s_1$

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source detector
to max power - $dA_1 = dA_2$, find $m, s_1, f, d\Phi_d$.
can use either underfill/overfill eq.
Irradiance on det (or in image) - effect of lens
$E = \pi L \sin^2\theta_{\frac{1}{2}}$
$\theta_{\frac{1}{2}} < 2^\circ$ - can use differential area
$E = \frac{L dA_s dA_d}{2\pi r^2} = \pi L \left(\frac{R_s}{r}\right)^2 = \pi L \tan^2\theta_{\frac{1}{2}}$
$\text{error}(1^\circ) = \frac{\tan^2 1^\circ - \sin^2 1^\circ}{\sin^2 1^\circ} < 10^{-3}$
error $(3^\circ) = 0.003$

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scattering of light
simple case - lamb scatterer
$\Omega_{scat} = 2\pi s.r.$
-paper, walls, moon - diffuse scatterer.
L_s, dA_{scatt}
$R d\Phi_{in} = d\Phi_{scatt}$
diffuse scatterin coefficient.
$R_{moon} = 0.12$
$RE = M$
$R\pi L \sin^2\theta_{\frac{1}{2}} = \pi L_{scatt}$
$L_{scatt} = RL \sin^2\theta_{\frac{1}{2}}$
Scattering reduces Radiance.
Passive optical system
propagation, refraction, scattering, reflection, reansmission, (diffraction)
no passive optical system can increase basic radiation.

Integrating sphere

has 2 ports, shine light in 1 port.
hit dA_0, scatter to dA_1

calculate power. $d^2\Phi = L_{scatter} dA_0 \cos\theta A_1 \frac{\cos\theta}{d^2}$

Since $\frac{d}{2} = R\cos\theta$,

$d^2\Phi = \frac{L_{scatt} dA_0 dA_1}{4R^2}$ (indep. of θ)

after 1 scattering,
r is scattering coefficient

$E_1 = \frac{\Phi_0 r}{4\pi R^2}$

$E_2 = \frac{\Phi_0}{4\pi R^2 (r + r^2)}$

$E = \frac{\Phi_0}{4\pi R^2} \frac{r}{1-r}$

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why use integrating sphere? If the power distribution of the input power in the transverse is very messy. The output of the power is uniform.

$i[A] = R\left[\frac{A}{W}\right]\Phi[W]$

purpose - homogenize the beam, and catching all $2\pi sr.$ of input.
large source, large pupil.
source standopint: $d^2\Phi = L dA_{s\perp} d\Omega_s$

$\Phi = \frac{L}{n^2} \cdot n^2 \int_{\text{solid angle source c}} \int_{\text{projected A}} dA_{s\perp} d\Omega_s$

pupil standpoint: $\Phi = \frac{L}{n^2} \cdot n^2 \int_{\text{solid angle seen by pupil}} \int_{\text{projected A of p}} dA_{p\perp} d\Omega_p$

called etendie
Conservation of Etendue
- ideal optical (no loss) conserves etendue
- no optical system can increase etendue.

Photometry - methods for calculating brightness (to human eye)
Lamb source, lens in front of eyeball (at the tip). Re retina
assume area of image > area of detector (overfill)
pupil of eye enlarge when dark.
material in eye with $n \neq 1$
$n_{eye} = 1.336$
brightness $\propto$ size of output of photoreceptor.
(angular) size = number of illuminated photoreceptors.
$i = R(\lambda)\Phi_\lambda(\lambda) d\lambda$
(overfill) $\Phi_\lambda(\lambda) = E_\lambda(\lambda) A_d = \pi \underbrace{L_{eye}}_{n^2_{eye} L_s} \sin^2\theta_{eye} A_d$

$i = \int R_i \pi n^2_{eye} \sin^2\theta_{eye} A_d L_{s\lambda}(\lambda) d\lambda$

brightness $= \int K_\lambda(\lambda) L_{s\lambda}(\lambda d\lambda$

$K_\lambda(\lambda)$ - spectral luminous efficacy.
numerical integral.
lumen per watt - unit of $K_\lambda(\lambda)$
peak at 555 nm.
ends of spectrum: $(400nm, 700nm)$
normalized version: $V_\lambda(\lambda)$

$K_\lambda(\lambda) = K_m V_\lambda(\lambda)$

brightness $= L_V = \int K_m V_\lambda(\lambda) L_{s,\lambda}(\lambda) d\lambda$

Luminence = brightness (only for extended source)

so far we considedered normal light adapted vision (photopic vision)

Dark adopted vision (scotopic vision)

the spectral luminence efficency distribution is slightly diff for dark adopted vision (peak at 510 nm))

$$L_V = K_m \int V_\lambda(\lambda) L_\lambda(\lambda) d\lambda = K_m(\Delta\lambda) \sum_{i=1}^{30} V_\lambda(\lambda) L_\lambda(\lambda) = K_m V_\lambda(\lambda) L$$

narrowband ("monochromatic") source

$$L_v \left[\frac{lm}{m^2 \cdot sr.}\right] = K_m \left[\frac{lm}{W}\right] \cdot V_\lambda(\lambda) \cdot L \left[\frac{W}{m^2 \cdot sr.}\right]$$

narrow band $L_V = K_m V_\lambda(\lambda) L$

unit is the same as $\left[\frac{cd}{m^2}\right], [nt]$

candida (cd) $= \frac{lm}{sr}$

nit (nt) $= \frac{Cd}{m^2}$

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ideal thermal emitter of light = Blackbody

Blackbody - an object that abosrbs all incident light (at all λ)

emission spectrum

y-axis - L_λ, M_λ or Φ_λ

asymmetric, long tail to the right

1D cavity: modes of cavity - specific wavelengths that constructively interfere.

$\lambda = 2l, l, \frac{2}{3}l, ... =$

$k = \frac{n\pi}{l}$ where $n \in \mathbb{N}$

3D: $\mathbf{k} = \frac{n_x\pi}{l_x}\hat{x} + \frac{n_y\pi}{l_y}\hat{y} + \frac{n_z\pi}{l_z}\hat{z}$

$$\rho_k = \frac{\text{\# of modes with wave vector } \mathbf{k}}{\text{Volume}} = \frac{1}{V}\frac{dN}{dk}$$

where $n = \#$ of modes

for $k \gg \frac{\pi}{l}$

k space volume of eighth shell/ k space volume per dot. $= \frac{\frac{1}{8} \cdot 4\pi k^2 dk}{\left(\frac{\pi}{l}\right)^2} = \frac{l^3 k^2 dk}{2\pi^3} = \frac{V k^2 dk}{2\pi^3}$

$$dN = \frac{V k^2 dk}{\pi^2}$$

$$\rho_k = \frac{1}{V}\frac{dN}{dk} = \frac{k^2}{\pi^2}$$

$$\rho_\nu = \rho_k \left|\frac{dk}{d\nu}\right| = \frac{k^2}{\pi^2}\frac{2\pi}{c} = \frac{8\pi\nu^2}{c^3}$$

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wrong version - Rayleigh -Jeans Law

$$u_\nu = \rho_\nu \cdot \frac{E}{\text{mode}} = \rho_\nu kT = \frac{8\pi\nu^2}{c^3}kT$$

$$\frac{E}{\text{mode}} = \overline{n} \cdot \underbrace{\text{energy of photon}}_{h\nu}$$

where $\overline{n}$ = mean # of photons per mode

equilibrium - prob. of a system in thermal equilibrium having energy E is $\propto e^{-\frac{E}{kT}}$

prob of a mode of freq ν having n photons $p(n) = Ae^{-\frac{nh\nu}{kT}}$

$$\sum_{n=0}^{\infty} p(n) = 1$$

$A = \frac{1}{1 - e^{-\frac{h\nu}{kT}}}$

$\overline{n} = \sum_{n=0}^{\infty} np(n) = \sum_{n=0}^{\infty} n \frac{e^{-\frac{nh\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}} = \frac{1}{1 - e^{-\frac{h\nu}{kT}}} \left(\frac{e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}} \right) = \frac{1}{e^{\frac{h\nu}{kT}} - 1}$

$u_\nu(\nu) = \rho_\nu(\nu)\overline{n}h\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$

$\frac{\text{mean energy}}{\text{mode}} = \overline{n}h\nu = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$

$\lim_{\nu \to 0} \overline{n}h\nu = kT$

d^3Q = energy flowing area dA from dΩ

$du = \frac{d^3Q}{dA \cdot \cos\theta \cdot c \cdot dt}$

$\frac{d^3Q}{dt} = d^2\Phi$

$\frac{d^2\Phi}{dA \cdot \cos\theta \cdot c}$

$u = \frac{4\pi L}{c}$

$L = \frac{uc}{4\pi}$

$L_V = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$

$$\text{brightness} = \begin{cases} \int R(\lambda)\Phi_{d,\lambda}(\lambda)d\lambda & \text{un-resolvable} \\ \int \underbrace{K_\lambda(\lambda)}_{K_m V_\lambda(\lambda)} L_\lambda(\lambda)d\lambda & \text{resolvable} \end{cases}$$

where $K_\lambda(\lambda) = R_i \pi n_{eye}^2 \sin^2\theta_{eye}$ (*)

Unresolvable: (underfill $h_i < h_d$ (detector size limited))

differential limited spot size (lens limited)

$\int R(\lambda)\pi L_\lambda(\lambda) \underbrace{\sin^2\theta_1}_{\frac{(\phi/2)^2}{s_1^2}} A_s d\lambda$

Narrow band: $L_V = K_m V_\lambda(\lambda) L$

---

ideal thermal emitter of light = Blackbody

Blackbody - an object that abosrbs all incident light (at all λ)

emission spectrum

y-axis - L_λ, M_λ or Φ_λ

asymmetric, long tail to the right

1D cavity: modes of cavity - specific wavelengths that constructively interfere.

$\lambda = 2l, l, \frac{2}{3}l, ... =$

$k = \frac{n\pi}{l}$ where $n \in \mathbb{N}$

3D: $\mathbf{k} = \frac{n_x\pi}{l_x}\hat{x} + \frac{n_y\pi}{l_y}\hat{y} + \frac{n_z\pi}{l_z}\hat{z}$

$\rho_k = \frac{\text{\# of modes with wave vector } \mathbf{k}}{\text{Volume}} = \frac{1}{V}\frac{dN}{dk}$

where n = # of modes

for $k \gg \frac{\pi}{l}$

k space volume of eighth shell/ k space volume per dot. $= \frac{\frac{1}{8} \cdot 4\pi k^2 dk}{\left(\frac{\pi}{l}\right)^2} = \frac{l^3k^2dk}{2\pi^3} = \frac{Vk^2dk}{2\pi^3}$

$dN = \frac{Vk^2dk}{\pi^2}$

$\rho_k = \frac{1}{V}\frac{dN}{dk} = \frac{k^2}{\pi^2}$

$\rho_\nu = \rho_k \left|\frac{dk}{d\nu}\right| = \frac{k^2}{\pi^2}\frac{2\pi}{c} = \frac{8\pi\nu^2}{c^3}$

---

wrong version - Rayleigh -Jeans Law

$u_\nu = \rho_\nu \cdot \frac{E}{\text{mode}} = \rho_\nu kT = \frac{8\pi\nu^2}{c^3}kT$

$\frac{E}{\text{mode}} = \overline{n} \cdot \underbrace{\text{energy of photon}}_{h\nu}$

where $\overline{n}$ = mean # of photons per mode

equilibrium - prob. of a system in thermal equilibrium having energy E is $\propto e^{-\frac{E}{kT}}$

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d^3Q = energy flowing area dA from dΩ

wrong $du = \frac{d^3Q}{dA\cdot\cos\theta\cdot c\cdot dt}$

$du = \frac{d^3Q}{dA\cdot\cos\theta d\Omega}\frac{d\Omega}{c}$

$du = L\frac{d\Omega}{c}$

$\frac{d^3Q}{dt} = d^2\Phi$

$\frac{d^2\Phi}{dA\cdot\cos\theta\cdot c}$

$u = \frac{4\pi L}{c}$

$L = \frac{uc}{4\pi}$

$L_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1} \qquad M_\nu = \pi L_\nu$

$M = \pi L = \pi\int_0^\infty L_\nu d\nu = \frac{\pi 2hn^2}{c^2}\int_0^\infty \frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}d\nu = n^2\left(\frac{2\pi^5k^4}{15h^3c^2}\right)T^4 = \sigma T^4$

$\sigma = 5.67\cdot 10^{-8}\frac{W}{m^2K^4}$

Stefan-Boltzman Law $L_\lambda = L_\nu\left|\frac{d\nu}{d\lambda}\right| = L\nu\frac{c}{n\lambda^2}$

$L_\lambda = \frac{2hc^2}{n\lambda^5}\frac{1}{e^{\frac{h\nu}{kT}}-1} \qquad M_\lambda = \pi L_\lambda$

finding max of L_λ : $\frac{dL_\lambda}{d\lambda} = 0 \to \lambda_{max} \approx \frac{2898\mu mK}{T}$

$\nu_{max} = 5.78 \cdot 10^{10} \frac{Hz}{K} T$
$L_\lambda(\lambda, T) > 0$
$T_1 < T_2 \Rightarrow L_\lambda(\lambda_0, T_1) < L_\lambda(\lambda_0, T_2)$

Photon-based radiaometric Quantities of BB
$u_{p,\nu} = \frac{8\pi v^2}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$
$L_{p,\nu} = \frac{2\nu^2}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$
$M_{p,\nu} = \frac{2\pi\nu^2}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$
$L_{p,\lambda} = \frac{2c}{\lambda^4} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$
$M_{p,\nu} = \frac{2\pi c}{\lambda^4} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$
$\nu^p_{max} = 3.315 \cdot 10^{10} \frac{Hz}{K} T$
$\lambda^p_{max} = \frac{3675 \mu m K}{T}$

$\epsilon = \epsilon(\lambda, \theta, \hat{\epsilon})$
$\epsilon = \frac{\Phi^{real}_{em}}{\Phi^{BB}_{em}} = \frac{M^{real}_{em}}{M^{BB}_{em}} = \frac{L^{real}_{em}}{L^{BB}_{em}}$
if ϵ known, $L^{real}_\lambda(\lambda) = \epsilon(\lambda) L^{BB}_\lambda(\lambda)$
$\alpha \equiv \frac{\Phi_{abs}}{\Phi_{in}}$

Absorption, Emission Equilibrium
$\Phi = VI$
$1 = T + R + \alpha$

---

steady state $\Phi_{em} = \Phi_{abs}$
Kirchoff's law (of radiation)
$\alpha \Phi_{in} = \Phi^{real}_{abs}$
$\alpha = \epsilon$
BB - perfect absorber, perfect emittedr
good reflector - metal mirror (R high, T=0 , $\epsilon = 1 - R$)
Good transmitter (glass) : $(T \approx 90\%, R = \text{few}\%, \epsilon = 1 - R - T)$
good absorber ($R = \text{low}, T = 0, \epsilon = \text{high}$)
BBs in lab: hole is nearly BB, temp controlled walls
the wlls are dull, black (low R)
purpose of BB - emit a known spectrum
calibration of spectral response of psectrumeter.

---

readiative heating
equilibrium $\Phi_{abs} = \Phi_{em}$
$\Phi_{em} = \Phi^{rad}_{em} + \Phi^{cond}_{em} + \Phi^{conv}_{em}$
if $\Phi^{cond}_{em}, \Phi^{conv}_{em}$
ex: thermal detector, isolated objects - earth
$\Phi_{abs} = \Phi^{rad}_{em}$
$\alpha \Phi_{in} = \Phi^{rad}_{em}$
$\Phi_{in} = \underbrace{\pi L_s}_{\sigma T_s^4} \sin^2 \theta_{\frac{1}{2}} A = \sigma T_d^4 A$

$\Rightarrow T_s\sqrt{\sin\theta_{\frac{1}{2}}} = T_d$
real body: $T_s\epsilon_s^{\frac{1}{4}}\sqrt{\sin\theta_{\frac{1}{2}}} = T_d$
$T_s > T_d$

---

earth heating by sun
Assumes Sun is BB, T = $57000K, \theta_{\frac{1}{2}} = 0.25°$
$\epsilon_{earth} = k$
$\alpha_{earth}\pi L_s \sin^2\theta_{\frac{1}{2}} A = \epsilon_{earth}\sigma T_e^4 A$
factor$^{\frac{1}{4}}\sqrt{\sin\theta_{\frac{1}{2}}}T_S = T_e$
abosrb during daytime, always emitting.
$T_e : 376K \to 316K = 45°$ by the factor$^{\frac{1}{4}}$
correct for angle of incoming sunlight.
factor $= \frac{1}{2}\cos\theta'$
θ' depends on season.
March - Sept: $T_e = 16°$
$37°$
$-27°$

$\Phi_{abs} = \Phi_{em}$
powers can be optical (Radiation), heat conduction, convection, electric
$\Phi_{abs}^{rad} = \epsilon\sigma T^4 A$ (in?)
$T = \left(\frac{\Phi_{abs}^{rad}}{\sigma A}\right)^{\frac{1}{4}}$
$\Phi_s + \Phi_{back} = T$ det
electricity
$\Delta V = G = i = \frac{dq}{dt}$
heat flow: $\frac{dQ}{dt} = K\Delta T$
$P_{heat} = K\Delta T$
$\epsilon\Phi_{bac} = \epsilon\sigma T_1^4 A$
$\epsilon\Phi_s + \epsilon\Phi_{back} = \epsilon\sigma T_2^4 A + K((T_2 - T_1)$
$\epsilon\Phi_s = \epsilon\sigma A(T_2^4 - T_1^4) + K(T_2 - T_1)$
small $\Delta T : \Delta(T^4) = 4T^3\Delta T$
solving for ΔT: $= \frac{\Phi_s}{4\sigma AT^3 + \frac{K}{\epsilon}}$
small $4\sigma AT^3$ (ideal limit) and small $\frac{K}{\epsilon}$
real thermal detector $\frac{K}{\epsilon} >> 4\sigma AT^3$
$\Phi_{abs} = iV = P_{elec}$
$\Phi_{em} = \Phi_{em}^{rad} = \epsilon\sigma T^4 A$
set equal: $P_{elec} = \epsilon\sigma T^4 A \Rightarrow T = \left(\frac{P_{elec}}{\epsilon\sigma A}\right)^{\frac{1}{4}}$
know $T \Rightarrow$ emitted spectrum is $\epsilon L_\lambda(\lambda T) \Rightarrow L_v, \Phi_v$, color

---

Colorimetry
iff only interested in monochromatic (narrowband) light : $\Delta\lambda \ll 300nm$
assign a color (or, more corrected, hue) according to its λ monochromatic light

$\lambda(nm)$	hue
<450	violent
450-490	blue
490-560	green
560-590	yellow
590-630	orangle
>630	red

broadband source?
Color atching experiments
3 specrtrum of color (call R,G,B).
if hae some color source, shine on screen.
Then shine the 3 RGB (alter the scales to match the color).
$L_\lambda^{(Q)}(\lambda) \sim RL_\lambda^{(R)}(\lambda) + GL_\lambda^{(G)}(\lambda) + BL_\lambda^{(B)}(\lambda)$
sometimes $Q = R + G + B$ could be found.
when no matcch could be found , sometimes a match like this could be found .
being one of the primary color over to Q.: $Q + B = R + G$
always exist a match.
$Q = \pm R \pm G \pm B$

---

further, 1 or 2 primary sometimes work.
4 or more primaries always produces a match, but not unique.
2 sources, with different $L_\lambda(\lambda)$
that appear the same to the eye are called metamers.
3 kind of color sensor in the eye (cones) 3 cones output 3 nu bers

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3-cones - detector
output current $= \int R_i(\lambda)\Phi_{\lambda,d}(\lambda)d\lambda \qquad R\left[\frac{A}{W}\right]$
$= c_0 \int R_i(\lambda)L_\lambda(\lambda)d\lambda$
red: $i_r = c_1 \int R_r(\lambda)L_\lambda(\lambda)d\lambda$
green: $i_g = c_2 \int R_g(\lambda)L_\lambda(\lambda)d\lambda$
Blue: $i_b = c_3 \int R_b(\lambda)L_\lambda(\lambda d\lambda$
colorimetry system
1 - define primaries - convention
a. use monochromatic sources .
b. choose radiance of primaries L^R, L^G, L^B s othat equal amount of the 3 primaries, ie., (equal tristimulis values) , we obtain a mixture that matches a spectrum that is called equal energy white (S_E)
$L_\lambda^{S_E}(\lambda) = \text{const}.$
ulus values:
a - use experiement
b- calculation (use color matching functions)
$\overline{r}(\lambda), \overline{g}(\lambda), \overline{b}(\lambda)$
if Q is monocromatic
$R = L^{(Q)}\overline{r}(\lambda_Q)$
$G = L^{(Q)}\overline{g}(\lambda_Q)$
$B = L^{(Q)}\overline{b}(\lambda_Q)$
color mathin function depends on primary
If Q is broadband
$R = \text{const} \int \overline{r}(\lambda)L_\lambda^{(Q)}(\lambda)d\lambda$

$G = \text{const} \int \overline{g}(\lambda) L_\lambda^{(Q)}(\lambda) d\lambda$
$B = \text{const} \int \overline{b}(\lambda) L_\lambda^{(Q)}(\lambda) d\lambda$
Consider test color, Q, that is just the 530 nm primary (G).
3 - at this point we have 3 parametrics (R, B, G) that specify 'appearance' of light.
;appearance' - brightness (photometry + color
color parametrers (indep. of brightness)
consider 2 test sources $Q)_1, Q_2$
suppose $L^{Q2} = nL^{Q1}, L_v^{Q2} = nL_v^{Q1}$
'color' of $Q2 = Q1$
$nR_1 = R_2$ (G, B too)
Chromaticity coordinates
$r = \frac{R}{R+G+B}, g = \frac{G}{R+G+B}, b = \frac{B}{R+G+B}$
$r_1 = r_2$ (g,b too)
indep. of brightness
$r + g + b = 1$
usully work with r,g to specify collor (chromaticity)
L_v specify brightness
appearance: r, g, L_v
3 sources corresponds to 3 points on the chromaticity diagram. the real maxiture of 2 sources form a line.
3 sources form an area, "gamut" the triangle .
'gamut' of all perceivable color (the whole diagram)
all monochromatic sources are on the curve arc - arc of spectrum
purple line (mixture of violet and red)

R, G, B - tristimulus values
$R_r(\lambda), R_g(\lambda), R_b(\lambda)$ - core response curves
$\overline{r}(\lambda), \overline{g}(\lambda), \overline{b}(\lambda)$ - color matching functions
$r = \frac{R}{R+G+B}, g, b$ - chromaticity coordinates (normalized tristumulus values).
2 stop process for determining chromaticity coord's
1 determine tristimulus values
use color matching function
ex. $R = \text{const} \cdot \int \overline{r}(\lambda) L_\lambda^{(Q)}(\lambda) d\lambda$
2 points on chromaticity diagram, outside of the line is imagery mixture
gamut of RGB primaries < gamut of all perceivable colors.
white some where in the middle.
brightness of G primary ¿ brightness of R primary.
two primaries X, Z ill have $L_v = 0$

---

imagery primary X, Y, Z, enclose all preceivable color (diagram 7)
S_E has coordinate $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
another white $c = (0.31, 0.316)$. ("average daylight")
blackbody: $T = 6750K$
for color Q, draw line from c to Q to edge (dominant wavelength - λ)

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anchor point c (white)
source in the triangular regin has no dominant wavelngth.
(has no hue)
dominant wavelength, complementary wavelength (other end when line extended)
some color has dominant wavelength but not complementary .

saturation - purity - $\frac{\overline{CQ}}{\overline{C \to \lambda_d}}$
appearance description: $X, Y, Z : x, y, Y$(brightness)
hue, saturation, , L_v(or Y)
